Working with Combined Sewers

Geometric and Hydraulic Elements of Cleveland Egg-Shaped Sewers – Part 2

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ABSTRACT Wastewater professionals working with combined sewers often encounter a variety of non-circular sewer cross sections, including egg-shaped sewers. Equations are developed here to calculate the wetted area, wetted perimeter, and surface width for various Cleveland egg-shaped sewers flowing partially full and support a variety of hydraulic calculations.

Part 1 describes the geometry of No. 2 and No. 3 Cleveland egg-shaped sewers, and Part 2 describes the geometry of No. 4 through No. 20. These equations supplant traditional graphical approaches that use geometric and hydraulic elements curves in favor of equations better suited to Smart Sewer applications. The equations are validated and provide a concise reference for those working with egg-shaped sewers in combined sewers located in Cleveland, Ohio.

KEY WORDS Combined Sewer, Non-Circular Section, Cleveland Egg-Shaped Sewer

Introduction

Combined sewers are designed and constructed with a wide variety of sewer cross sections. While the circular section is most common, other non-circular sections have been used over the years.[1] Working with combined sewers requires an understanding of the geometric and hydraulic elements of these non-circular sections when encountered.

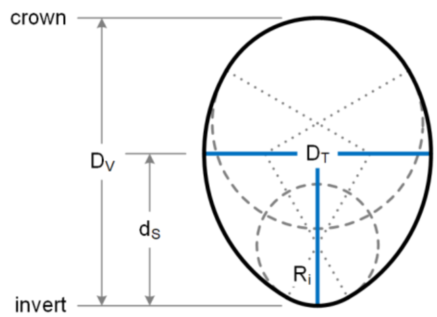
Geometric properties of non-circular sections are found in standard engineering references but are often presented only in graphical form for sewers flowing partially full and where the results are provided as a proportion of values for sewers flowing full.[1] While this approach was sufficient when these sewers were originally designed and constructed, it is insufficient for modern Smart Sewer applications where automated and precise computations are required.

Various egg-shaped sewer sections used in Cleveland, Ohio are the focus of this discussion. Descriptions of No. 4 through No. 20 Cleveland egg-shaped sewers are provided here, and historical perspectives regarding their development and application are presented. The geometric elements of these egg-shaped sections are then defined, and equations for the wetted area, wetted perimeter, and surface width for sewers flowing partially full are provided. The result is a concise reference for wastewater professionals working with egg-shaped sewers in combined sewers located in Cleveland, Ohio.

Description

While various egg-shaped sewers have been developed and used over the years, several unique versions are found in Cleveland, Ohio, one of which is shown in Figure 1. The geometry of this egg-shaped sewer – used for No. 4 through No. 20 Cleveland egg-shaped sewers – begins with the *invert radius* (R_i), and other relevant dimensions – including the transverse diameter (D_T) and vertical diameter (D_V) – are each defined as a function of R_i.

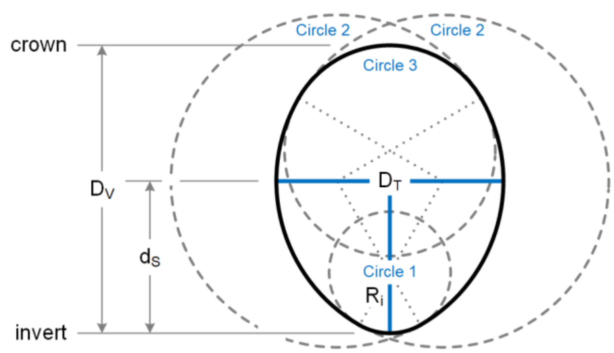
FIGURE 1: Cleveland Egg-Shaped Sewer Cross-Section



No. 4 – No. 20

The anatomy of No. 4 through No. 20 Cleveland egg-shaped sewers is shown below in Figure 2.

FIGURE 2: Anatomy of a Cleveland Egg-Shaped Sewer



No. 4 – No. 20

This egg-shaped sewer is a compound cross-section formed by several tangent circles. Circle 1 forms the invert of the egg-shaped sewer, which is positioned at a specified distance (d_S) from the invert to the spring line. Circle 2 forms the sides, and Circle 3 forms the arch. Relative dimensions of each component of these Cleveland egg-shaped sewers are provided in Table 1.

TABLE 1: Relative Component Dimensions of Cleveland Egg-Shaped Sewers

Component	No. 4 – No. 20	
	Diameter	Depth
Circle 1	$D_1 = 2R_i$	$d_1 = d$
Circle 2	$D_2 = 2R_i(1+\sqrt{3})$	$d_2 = d + R_i\left(\sqrt{3} - \frac{3}{2}\right)$
Circle 3	$D_3 = 2R_i\sqrt{3}$	$d_3 = d - R_i(3-\sqrt{3})$
Egg	$D_T = R_i(2+\sqrt{3})$	$d_S = \frac{5}{2}R_i$
	$D_V = R_i(3+\sqrt{3})$	

The diameter (D_1, D_2, D_3) of each component circle is defined relative to R_i as specified by Cyrus G. Force, Jr., and the flow depth in each component circle (d_1, d_2, d_3) is derived relative to the flow depth (d) in No. 4 through No. 20 Cleveland egg-shaped sewers.[2]

Historical Perspective

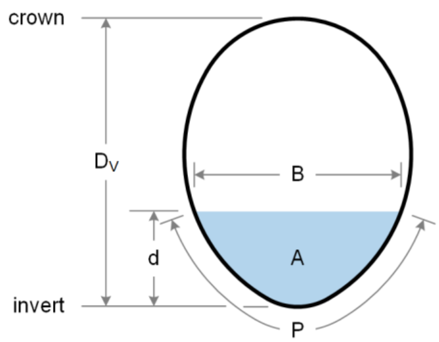
The egg-shaped sewer was developed in the 19th Century to enhance self-cleansing in combined sewers by channeling wastewater flows through a smaller cross-section during low flow conditions, resulting in a greater flow velocity when compared to a similarly sized circular sewer.[3] John Phillips (1817 – 1897) is associated with several egg-shaped sewer designs used in London, two of which became most popular.[4] These designs spread throughout the world and were adopted in many developing cities, including Cleveland, Ohio.[2]

While these egg-shaped sewers were successfully introduced in other cities, structural problems were encountered in Cleveland due to groundwater and unfavorable soil conditions.[2] Cyrus G. Force, Jr. (1841 – 1922), an engineer who worked for the City of Cleveland, designed an egg-shaped sewer section to overcome these difficulties and proposed several sizes designated No. 1 through No. 16.[2,5] The No. 1 Cleveland egg-shaped sewer was never adopted, and other egg-shaped sewer sections were implemented for No. 2 and No. 3 Cleveland egg-shaped sewers, designed by others.[2,6,7] No. 4 through No. 16 Cleveland egg-shaped sewers were adopted as designed by Cyrus G. Force, Jr. and were eventually extended to additional sizes, including No. 17 through No. 20 Cleveland egg-shaped sewers.[8] While unique to the City of Cleveland, these sewers have passed the test of time, with many of them in service for well over 100 years.

Geometric Elements

Several *geometric elements* are needed for engineering calculations involving sewers flowing partially full and are a function of flow depth (d), as well as the size and shape of the sewer cross section. The *wetted area* (A) is the part of the sewer cross section occupied by the wastewater flow, while the *wetted perimeter* (P) is the part of the sewer perimeter in contact with the wastewater flow.[9] The *surface width* (B) is the width of the flow surface.[9] The geometric elements of No. 4 through No. 20 Cleveland egg-shaped sewers are illustrated in Figure 3.

FIGURE 3: Geometric Elements of a Partially Filled Cleveland Egg-Shaped Sewer

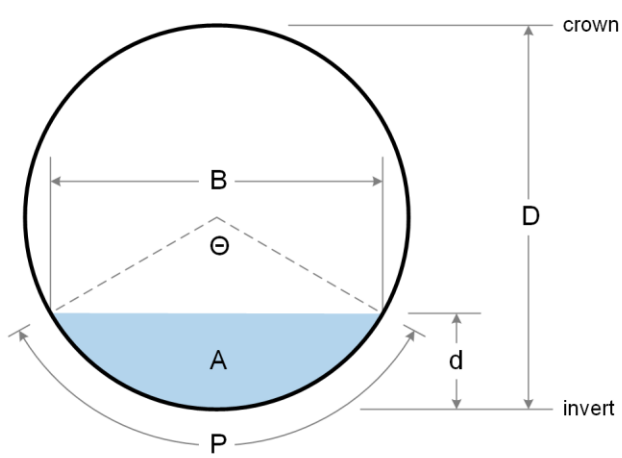


No. 4 – No. 20

Partially Filled Circular Sewer

Since an egg-shaped sewer is constructed from several circles, its geometric elements are based on the geometry of a partially filled circular sewer as illustrated in Figure 4 and defined in Equations (1) through (4).[9]

FIGURE 4: Geometric Elements of a Partially Filled Circle



$$\theta = 2\cos^{-1}\left(1 - 2\frac{d}{D}\right) \quad (1)$$

$$A = \frac{D^2}{8}(\theta - sin\theta) \quad (2)$$

$$P = \frac{D}{2}\theta \quad (3)$$

$$B = Dsin\frac{\theta}{2} \quad (4)$$

where:
θ = central angle, radians
d = flow depth, ft
D = diameter, ft
A = wetted area, ft^2
P = wetted perimeter, ft
B = surface width, ft

Partially Filled Cleveland Egg-Shaped Sewers – No. 4 Through No. 20

Once the geometry of No. 4 through No. 20 Cleveland egg-shaped sewers is defined, several regions are identified with distinct geometric calculations. Three regions within No. 4 through No. 20 Cleveland egg-shaped sewers are detailed in Table 2.

TABLE 2: Geometric Regions of Cleveland Egg-Shaped Sewers – No. 4 Through No. 20

Region	Depth
I	$0 \leq d \leq R_i\left(1-\frac{\sqrt{3}}{2}\right)$
II	$R_i\left(1-\frac{\sqrt{3}}{2}\right) < d \leq R_i\left(3+\frac{\sqrt{3}}{2}\right)$
III	$R_i\left(3+\frac{\sqrt{3}}{2}\right) < d \leq R_i\left(3+\sqrt{3}\right)$

Geometric elements in Region I are based on Circle 1. Geometric elements in Region II are based on Circle 1 and Circle 2, and geometric elements in Region III are based on Circle 1, Circle 2, and Circle 3.

The wetted area of a partially filled No. 4 through No. 20 Cleveland egg-shaped sewer is calculated using the compound equation shown in Equation (5).

$$A_{egg} = \tag{5}$$

when d is in Region I	when d is in Region II	when d is in Region III
$+A_1 \quad where\ d = d$	$+A_2 \quad where\ d = d$	$+A_3 \quad where\ d = d$
	$-A_2 \quad where\ d = R_i\left(1-\frac{\sqrt{3}}{2}\right)$	$-A_3 \quad where\ d = R_i\left(3+\frac{\sqrt{3}}{2}\right)$
	$-R_i\sqrt{3}\left[d - R_i\left(1-\frac{\sqrt{3}}{2}\right)\right] \quad where\ d = d$	$+A_2 \quad where\ d = R_i\left(3+\frac{\sqrt{3}}{2}\right)$
	$+A_1 \quad where\ d = R_i\left(1-\frac{\sqrt{3}}{2}\right)$	$-A_2 \quad where\ d = R_i\left(1-\frac{\sqrt{3}}{2}\right)$
		$-R_i^2\left(3+2\sqrt{3}\right)$
		$+A_1 \quad where\ d = R_i\left(1-\frac{\sqrt{3}}{2}\right)$

Circle 1	Circle 2	Circle 3
$\theta_1 = 2\cos^{-1}\left(1-\frac{d}{R_i}\right)$	$\theta_2 = 2\cos^{-1}\left[1-\frac{d+R_i\left(\sqrt{3}-\frac{3}{2}\right)}{R_i\left(1+\sqrt{3}\right)}\right]$	$\theta_3 = 2\cos^{-1}\left[1-\frac{d-R_i\left(3-\sqrt{3}\right)}{R_i\sqrt{3}}\right]$
$A_1 = \frac{R_i^2}{2}(\theta_1 - \sin\theta_1)$	$A_2 = R_i^2\left(2+\sqrt{3}\right)(\theta_2 - \sin\theta_2)$	$A_3 = \frac{3R_i}{2}(\theta_3 - \sin\theta_3)$

The wetted perimeter of a partially filled No. 4 through No. 20 Cleveland egg-shaped sewer is calculated using the compound equation shown in Equation (6).

$$P_{egg} = \tag{6}$$

$+P_1 \quad where\ d = d$ $when\ d\ is\ in\ Region\ I$	$+P_2 \quad where\ d = d$ $-P_2 \quad where\ d = R_i\left(1 - \frac{\sqrt{3}}{2}\right)$ $+P_1 \quad where\ d = R_i\left(1 - \frac{\sqrt{3}}{2}\right)$ $when\ d\ is\ in\ Region\ II$	$+P_3 \quad where\ d = d$ $-P_3 \quad where\ d = R_i\left(3 + \frac{\sqrt{3}}{2}\right)$ $+P_2 \quad where\ d = R_i\left(3 + \frac{\sqrt{3}}{2}\right)$ $-P_2 \quad where\ d = R_i\left(1 - \frac{\sqrt{3}}{2}\right)$ $+P_1 \quad where\ d = R_i\left(1 - \frac{\sqrt{3}}{2}\right)$ $when\ d\ is\ in\ Region\ III$
Circle 1 $\theta_1 = 2\cos^{-1}\left(1 - \frac{d}{R_i}\right)$ $P_1 = R_i\theta_1$	Circle 2 $\theta_2 = 2\cos^{-1}\left[1 - \frac{d + R_i\left(\sqrt{3} - \frac{3}{2}\right)}{R_i(1+\sqrt{3})}\right]$ $P_2 = R_i(1+\sqrt{3})\theta_2$	Circle 3 $\theta_3 = 2\cos^{-1}\left[1 - \frac{d - R_i(3-\sqrt{3})}{R_i\sqrt{3}}\right]$ $P_3 = R_i\sqrt{3}\theta_3$

The surface width of a partially filled No. 4 through No. 20 Cleveland egg-shaped sewer is calculated using the compound equation shown in Equation (7).

$$B_{egg} = \tag{7}$$

$+B_1 \quad where\ d = d$ $when\ d\ is\ in\ Region\ I$	$+B_2 \quad where\ d = d$ $-R_i\sqrt{3}$ $when\ d\ is\ in\ Region\ II$	$+B_3 \quad where\ d = d$ $when\ d\ is\ in\ Region\ III$
Circle 1 $\theta_1 = 2\cos^{-1}\left(1 - \frac{d}{R_i}\right)$ $B_1 = 2R_i\sin\frac{\theta_1}{2}$	Circle 2 $\theta_2 = 2\cos^{-1}\left[1 - \frac{d + R_i\left(\sqrt{3} - \frac{3}{2}\right)}{R_i(1+\sqrt{3})}\right]$ $B_2 = 2R_i(1+\sqrt{3})\sin\frac{\theta_2}{2}$	Circle 3 $\theta_3 = 2\cos^{-1}\left[1 - \frac{d - R_i(3-\sqrt{3})}{R_i\sqrt{3}}\right]$ $B_3 = 2R_i\sqrt{3}\sin\frac{\theta_3}{2}$

The wetted area and wetted perimeter of No. 4 through No. 20 Cleveland egg-shaped sewers *when flowing full* are calculated using Equation (8) and Equation (9), respectively.[2]

$$A_{egg} = R_i^2 \left[\pi \left(\frac{19}{6} + \sqrt{3} \right) - \sqrt{3} \right] \tag{8}$$

$$P_{egg} = \frac{\pi R_i}{3} \left(4 + 5\sqrt{3} \right) \tag{9}$$

where: A_{egg} = wetted area, ft²
P_{egg} = wetted perimeter, ft
R_i = invert radius, ft

These equations were developed by Cyrus G. Force, Jr. and represent special cases of Equation (5) and Equation (6) under flow pipe flow conditions.[2]

Designated Sizes of Cleveland Egg-Shaped Sewers

Based on this egg-shaped sewer section, Cyrus G. Force, Jr. originally designated 16 sizes – No. 1 through No. 16 – where the full-pipe wetted area of each size is specified using an assigned formula as shown in Equation (10).[2]

$$A_{egg} = \sqrt[3]{N^4} \tag{10}$$

where: A_{egg} = wetted area, ft²
N = designated size, No. N

The invert radius (R_i) is then calculated for each No. N by substituting Equation (10) into Equation (8) and solving for R_i as shown in Equation (11).

$$R_i = \sqrt{\frac{\sqrt[3]{N^4}}{\pi \left(\frac{19}{6} + \sqrt{3} \right) - \sqrt{3}}} \tag{11}$$

where: R_i = invert radius, ft
N = designated size, No. N

Note that this cross-section and these formulas do not apply to No. 2 and No. 3 Cleveland egg-shaped sewers. They only apply to No. 4 through No. 20 Cleveland egg-shaped sewers. The dimensions of these sewers are shown in Table 3.

TABLE 3: Dimensions of Cleveland Egg-Shaped Sewers

No. N	R_i (ft)	D_T (ft)	D_V (ft)	d_S (ft)	A_{egg} (ft^2)	P_{egg} (ft)
4	0.6818	2.5447	3.2265	1.7046	6.3496	9.0397
5	0.7912	2.9528	3.7440	1.9780	8.5499	10.4897
6	0.8935	3.3345	4.2279	2.2337	10.9027	11.8454
7	0.9902	3.6954	4.6855	2.4754	13.3905	13.1275
8	1.0824	4.0394	5.1218	2.7059	16.0000	14.3497
9	1.1708	4.3694	5.5402	2.9269	18.7208	15.5219
10	1.2560	4.6873	5.9433	3.1399	21.5443	16.6513
11	1.3384	4.9948	6.3332	3.3459	24.4638	17.7437
12	1.4183	5.2931	6.7114	3.5457	27.4731	18.8034
13	1.4960	5.5833	7.0793	3.7401	30.5674	19.8340
14	1.5718	5.8660	7.4378	3.9295	33.7420	20.8385
15	1.6458	6.1421	7.7879	4.1144	36.9932	21.8194
16	1.7181	6.4122	8.1303	4.2953	40.3175	22.7787
17	1.7890	6.6766	8.4656	4.4725	43.7118	23.7182
18	1.8585	6.9360	8.7944	4.6462	47.1733	24.6394
19	1.9267	7.1905	9.1172	4.8167	50.6996	25.5437
20	1.9937	7.4407	9.4344	4.9843	54.2884	26.4323

Validation

Equations (5) through (7) are validated by computing the wetted area, wetted perimeter, and surface width at various flow depths for No. 4 through No. 20 Cleveland egg-shaped sewers and comparing the results to reference equations and/or values derived from them. Results are provided in Table 4 through Table 20 for No. 4 through No. 20 Cleveland egg-shaped sewers, respectively. The flow depths chosen for comparison include the maximum flow depth in Regions I, II, and III.

TABLE 4: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 4

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5)	Ref.	Δ	Eqn. (6)	Ref.	Δ	Eqn. (7)	Ref.	Δ
	(ft^2)	(ft^2)	(%)	(ft)	(ft)	(%)	(ft)	(ft)	(%)
I	0.0421	0.0421	0%	0.7140	0.7140	0%	0.6818	0.6818	0%
II	5.4930	5.4930	0%	6.5663	6.5663	0%	2.0455	2.0455	0%
III	6.3496	6.3496	0%	9.0397	9.0397	0%	0.0000	0.0000	0%

TABLE 5: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 5

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5)	Ref.	Δ	Eqn. (6)	Ref.	Δ	Eqn. (7)	Ref.	Δ
	(ft^2)	(ft^2)	(%)	(ft)	(ft)	(%)	(ft)	(ft)	(%)
I	0.0567	0.0567	0%	0.8286	0.8286	0%	0.7912	0.7912	0%
II	7.3964	7.3964	0%	7.6195	7.6195	0%	2.3736	2.3736	0%
III	8.5499	8.5499	0%	10.4897	10.4897	0%	0.0000	0.0000	0%

TABLE 6: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 6

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5)	Ref.	Δ	Eqn. (6)	Ref.	Δ	Eqn. (7)	Ref.	Δ
	(ft^2)	(ft^2)	(%)	(ft)	(ft)	(%)	(ft)	(ft)	(%)
I	0.0723	0.0723	0%	0.9356	0.9356	0%	0.8935	0.8935	0%
II	9.4318	9.4318	0%	8.6043	8.6043	0%	2.6804	2.6804	0%
III	10.9027	10.9027	0%	11.8454	11.8454	0%	0.0000	0.0000	0%

TABLE 7: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 7

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5)	Ref.	Δ	Eqn. (6)	Ref.	Δ	Eqn. (7)	Ref.	Δ
	(ft^2)	(ft^2)	(%)	(ft)	(ft)	(%)	(ft)	(ft)	(%)
I	0.0888	0.0888	0%	1.0369	1.0369	0%	0.9902	0.9902	0%
II	11.5840	11.5840	0%	9.5355	9.5355	0%	2.9705	2.9705	0%
III	13.3905	13.3905	0%	13.1275	13.1275	0%	0.0000	0.0000	0%

TABLE 8: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 8

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.1061	0.1061	0%	1.1334	1.1334	0%	1.0824	1.0824	0%
II	13.8415	13.8415	0%	10.4233	10.4233	0%	3.2471	3.2471	0%
III	16.0000	16.0000	0%	14.3497	14.3497	0%	0.0000	0.0000	0%

TABLE 9: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 9

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.1242	0.1242	0%	1.2260	1.2260	0%	1.1708	1.1708	0%
II	16.1951	16.1951	0%	11.2748	11.2748	0%	3.5123	3.5123	0%
III	18.7208	18.7208	0%	15.5219	15.5219	0%	0.0000	0.0000	0%

TABLE 10: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 10

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.1429	0.1429	0%	1.3152	1.3152	0%	1.2560	1.2560	0%
II	18.6378	18.6378	0%	12.0952	12.0952	0%	3.7679	3.7679	0%
III	21.5443	21.5443	0%	16.6513	16.6513	0%	0.0000	0.0000	0%

TABLE 11: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 11

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.1623	0.1623	0%	1.4015	1.4015	0%	1.3384	1.3384	0%
II	21.1634	21.1634	0%	12.8886	12.8886	0%	4.0151	4.0151	0%
III	24.4638	24.4638	0%	17.7437	17.7437	0%	0.0000	0.0000	0%

TABLE 12: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 12

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.1822	0.1822	0%	1.4852	1.4852	0%	1.4183	1.4183	0%
II	23.7668	23.7668	0%	13.6584	13.6584	0%	4.2549	4.2549	0%
III	27.4731	27.4731	0%	18.8034	18.8034	0%	0.0000	0.0000	0%

TABLE 13: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 13

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.2027	0.2027	0%	1.5666	1.5666	0%	1.4960	1.4960	0%
II	26.4435	26.4435	0%	14.4070	14.4070	0%	4.4881	4.4881	0%
III	30.5674	30.5674	0%	19.8340	19.8340	0%	0.0000	0.0000	0%

TABLE 14: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 14

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.2238	0.2238	0%	1.6460	1.6460	0%	1.5718	1.5718	0%
II	29.1899	29.1899	0%	15.1367	15.1367	0%	4.7154	4.7154	0%
III	33.7420	33.7420	0%	20.8385	20.8385	0%	0.0000	0.0000	0%

TABLE 15: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 15

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.2454	0.2454	0%	1.7235	1.7235	0%	1.6458	1.6458	0%
II	32.0025	32.0025	0%	15.8492	15.8492	0%	4.9373	4.9373	0%
III	36.9932	36.9932	0%	21.8194	21.8194	0%	0.0000	0.0000	0%

TABLE 16: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 16

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.2674	0.2674	0%	1.7992	1.7992	0%	1.7181	1.7181	0%
II	34.8783	34.8783	0%	16.5460	16.5460	0%	5.1544	5.1544	0%
III	40.3175	40.3175	0%	22.7787	22.7787	0%	0.0000	0.0000	0%

TABLE 17: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 17

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.2899	0.2899	0%	1.8734	1.8734	0%	1.7890	1.7890	0%
II	37.8147	37.8147	0%	17.2284	17.2284	0%	5.3670	5.3670	0%
III	43.7118	43.7118	0%	23.7182	23.7182	0%	0.0000	0.0000	0%

TABLE 18: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 18

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.3129	0.3129	0%	1.9462	1.9462	0%	1.8585	1.8585	0%
II	40.8092	40.8092	0%	17.8976	17.8976	0%	5.5755	5.5755	0%
III	47.1733	47.1733	0%	24.6394	24.6394	0%	0.0000	0.0000	0%

TABLE 19: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 19

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.3363	0.3363	0%	2.0176	2.0176	0%	1.9267	1.9267	0%
II	43.8598	43.8598	0%	18.5544	18.5544	0%	5.7801	5.7801	0%
III	50.6996	50.6996	0%	25.5437	25.5437	0%	0.0000	0.0000	0%

TABLE 20: Validation of Equations for a Partially Filled Cleveland Egg-Shaped Sewer – No. 20

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (ft^2)	Ref. (ft^2)	Δ (%)	Eqn. (6) (ft)	Ref. (ft)	Δ (%)	Eqn. (7) (ft)	Ref. (ft)	Δ (%)
I	0.3601	0.3601	0%	2.0878	2.0878	0%	1.9937	1.9937	0%
II	46.9643	46.9643	0%	19.1999	19.1999	0%	5.9812	5.9812	0%
III	54.2884	54.2884	0%	26.4323	26.4323	0%	0.0000	0.0000	0%

At the maximum flow depth in Region I, the wetted area, wetted perimeter, and surface width are equivalent to those of Circle 1 at the specified flow depth. The values obtained from Equations (5) through (7) for No. 4 through No. 20 Cleveland egg-shaped sewers equal the expected values, as shown in Table 4 through Table 20, respectively.

At the maximum flow depth in Region II, no reference equations are available for comparison. However, the expected wetted area and wetted perimeter are derived by taking the full-pipe values from Equations (8) and (9) and subtracting the wetted area or wetted perimeter of Circle 3 when 75% full. While no reference equation is available for the surface width, the geometry of No. 4 through No. 20 Cleveland egg-shaped sewers dictates that it is equal to the surface width of Circle 3 when 75% full. The values obtained from Equations (5) through (7) for No. 4 through No. 20 Cleveland egg-shaped sewers equal these derived values and geometric expectations, as shown in Table 4 through Table 20, respectively.

At the maximum flow depth in Region III, Equations (8) and (9) are used to calculate the wetted area and wetted perimeter under full-flow conditions. While no reference equation is available for the surface width, the geometry of the Cleveland egg-shaped sewers dictates that it is zero at this flow depth. The values obtained from Equations (5) through (7) for No. 4 through No. 20 Cleveland egg-shaped sewers equal those obtained using reference equations and geometric expectations, as shown in Table 4 through Table 20, respectively.

Hydraulic Elements

Once the geometric elements of partially filled Cleveland egg-shaped sewers are known, several *hydraulic elements* needed for commonly used wastewater engineering calculations can be determined. The *hydraulic radius* (R) is calculated using Equation (12) and is defined as the ratio of the wetted area to the wetted perimeter.[9]

$$R = \frac{A}{P} \qquad (12)$$

where: R = hydraulic radius, ft
A = wetted area, ft^2
P = wetted perimeter, ft

The *hydraulic mean depth* (d_h) is calculated using Equation (13) and is defined as the ratio of the wetted area to the surface width.[9]

$$d_h = \frac{A}{B} \tag{13}$$

where: d_h = hydraulic mean depth, ft
A = wetted area, ft^2
B = surface width, ft

The hydraulic radius is used in the Manning Equation as shown in Equations (14) and (15) to estimate the flow velocity (v) and flow rate (Q) under uniform flow conditions.[1]

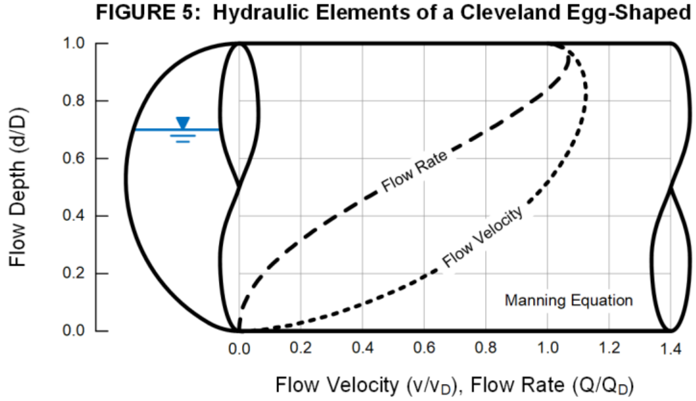
$$v = \frac{1.486}{n} R^{2/3} S^{1/2} \tag{14}$$

$$Q = \frac{1.486}{n} A R^{2/3} S^{1/2} \tag{15}$$

where: v = flow velocity, ft/s
n = roughness coefficient
R = hydraulic radius, ft
S = slope of the energy gradient
Q = flow rate, ft^3/s
A = wetted area, ft^2

Hydraulic elements curves prepared using the Manning Equation for No. 4 through No. 20 Cleveland egg-shaped sewers at various relative flow depths are shown in Figure 5.

FIGURE 5: Hydraulic Elements of a Cleveland Egg-Shaped Sewer



Other important properties enabled once the geometric elements of partially full Cleveland egg-shaped sewers are known include the Froude Number, Reynolds Number, and shear stress.[9,10]

Conclusion

Combined sewers are designed and constructed with a wide variety of sewer cross sections. While the circular section is most common, other non-circular sections have also been used over the years. Working with combined sewers requires an understanding of the geometric and hydraulic elements of these non-circular sections when encountered.

The geometric elements of No. 4 through No. 20 Cleveland egg-shaped sewers flowing partially full have been defined, and equations for the wetted area, wetted perimeter, and surface width are provided and validated. The result is a concise reference for wastewater professionals working with egg-shaped sewers in combined sewers located in Cleveland, Ohio.

Symbols and Notation

The following symbols and notation are used in this paper:

VARIABLES

θ = central angle, radians
d = flow depth, ft
D = diameter, ft
A = wetted area, ft^2
P = wetted perimeter, ft
B = surface width, ft
R = radius, ft
v = flow velocity, ft/s
Q = flow rate, ft^3/s
n = roughness coefficient
S = slope of the energy gradient

SUBSCRIPTS

$_T$ = transverse
$_V$ = vertical
$_S$ = spring line
$_1$ = Circle 1
$_2$ = Circle 2
$_3$ = Circle 3
$_i$ = invert
$_{egg}$ = egg
$_h$ = hydraulic

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The author acknowledges Cyrus G. Force, Jr. and his colleagues whose engineering creativity in the late 1800s led to the design and construction of these egg-shaped sewers unique to Cleveland, Ohio. That they accomplished these designs with a more rudimentary knowledge of sewer hydraulics known at the time and with computational tools available in their day is astounding. Special thanks are extended to the Northeast Ohio Regional Sewer District (NEORSD) who maintains many of these egg-shaped sewers to this day, as well as to the dedicated team at ADS Environmental Services who monitor flows within them and whose efforts were the inspiration for this work.

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